

SPECTRAL-DOMAIN ANALYSIS  
FOR DIELECTRIC ANTENNA LOADED WITH METALLIC STRIPS

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### ABSTRACT

A spectral-domain analysis is presented for a millimeter-wave antenna that consists of a dielectric rod loaded periodically with metallic strips. Numerical results are given for the leakage constant and the radiation angle over a large range of frequency. In particular, the effect of finite antenna width is examined in detail, with the polarization coupling between TE and TM modes properly taken into account in the analysis. Experiments are carried out in the X-band frequencies, and the results obtained verify the theory very well.

### INTRODUCTION

A dielectric rod loaded periodically with metal strips along its axis can be used for the design of antennas. Being a periodic structure, it belongs to the class of leaky-wave antennas which offer the capability of frequency scanning of the radiating beam. The structure is simple in geometry and easy to fabricate; more importantly, it is compatible with integrated-circuit (IC) technology for millimeter-wave applications.

Theoretical and experimental investigations of this antenna structure have been quite extensive in the literature[1-5]. While experimental results have demonstrated its usefulness, all the published theories had been carried out with the simplifying assumption that the antenna has an infinite width, and the excitation is invariant along the width. Under such a simplifying assumption, it is a two-dimensional scalar boundary value problem that supports the independent TE and TM modes. However, for millimeter-wave applications, the antenna width is usually of the order of a free-space wavelength and the simplifying assumption made in the past may not be generally valid. When the effect of finite antenna width is included in the analysis, it then becomes a three-dimensional vector boundary value problem that yields hybrid modes. This means that the constituent "TE" and "TM" guided modes are generally coupled within the antenna structure with a finite width[6,7]. The effect of such a polarization coupling remains to be quantified.

In this paper, a formulation of the antenna structure with a finite width is carried out by the method of spectral-domain analysis. The

hybrid nature of the guided mode is analyzed in terms of the coupling between constituent TE and TM guided modes at the geometrical discontinuities of the metal strip. Numerical results are obtained for various structure parameters. The effect of finite antenna width on the polarization coupling is examined. Finally, experimental results are obtained and they agree well with the theory.

### FORMULATION

To study the effect of finite antenna width, we analyze here the structure shown in Fig. 1. The dielectric rod with periodically loaded metal strips is placed in a trough waveguide, so that the field variation in the y-direction can be exactly and easily determined. The metal strips have the length b and the width w; its thickness is assumed vanishingly small.

By taking the y-axis as the direction of propagation, the structure can be divided into two uniform regions: the dielectric and air regions, which share the same set of transverse mode functions of the parallel-plate waveguide. The structure may support the TEM mode, with no variation in the y-direction, but this is not a mode of interest to us. We consider here only the lowest non-TEM mode. Because of the periodicity in the z-direction, the propagation constants of the space harmonics in that direction are:

$$k_{zn} = k_{zo} + 2n\pi/d \quad (1)$$

where d is the period. In the parallel-plate waveguide, each space harmonic propagates independently and the transverse components of the electric and magnetic fields (in the xz-plane) are written as the superpositions of the mode vectors, as:

$$\underline{E}_t = \sum_n [V'_n(y)\underline{e}'_n(x,z) + V''_n(y)\underline{e}''_n(x,z)] \quad (2)$$

$$\underline{H}_t = \sum_n [I'_n(y)\underline{h}'_n(x,z) + I''_n(y)\underline{h}''_n(x,z)] \quad (3)$$

where the single prime denotes the TE modes and the double prime denote the TM modes. The modal voltages and currents, V's and I's, and the mode vectors, e's and h's, are given by:

$$V'_n(y) = \begin{cases} A_n \cos(k_{yn}y), & \text{for } 0 < y < b \\ B_n \exp(-jk_{y2}y), & \text{for } y > b \end{cases} \quad (4)$$

$$V_n''(y) = \begin{cases} C_n \cos(k_{yn} y), & \text{for } 0 < y < b \\ D_n \exp(-jk_{y2} y), & \text{for } y > b \end{cases} \quad (5)$$

$$e_n' = y_0 x h_n' = V_t [\cos(k_x x) \exp(-jk_{zn} z)] \quad (6)$$

$$h_n'' = e_n'' x y_0 = V_t [\sin(k_x x) \exp(-jk_{zn} z)] \quad (7)$$

where  $A_n$ ,  $B_n$ ,  $C_n$  and  $D_n$  are unknowns to be determined by applying the continuity conditions at the interface  $y=b$ :

$$E_t(x, b^+, z) = E_t(x, b^-, z) \quad (8)$$

$$H_t(x, b^+, z) - H_t(x, b^-, z) = J_n \times y_0 \quad (9)$$

where  $J_n$  is the  $n$ -th space harmonic of the current vector on the metal strips, with the components:  $J_{xn}$  and  $J_{zn}$ .

Substituting (2) and (3) into (8) and (9), we obtain a set of linear algebraic equations for unknown coefficients  $A_n$ ,  $B_n$ ,  $C_n$  and  $D_n$ . After eliminating these unknown coefficients in the resultant equations, we then obtain:

$$G_{xx} J_{xn} + G_{xz} J_{zn} = E_{xn} \quad (10)$$

$$G_{xz} J_{xn} + G_{zz} J_{zn} = E_{zn} \quad (11)$$

where the  $G$ 's are defined by:

$$G_{xx} = (\omega u_0 / \Delta) (j(k_1^2 - k_x^2) \text{ctg}(k_{y1} b) / k_{y1} - (k_2^2 - k_x^2) / k_{y2}) \quad (12)$$

$$G_{xz} = -(\omega u_0 / \Delta) k_x k_z (\text{ctg}(k_{y1} b) / k_{y1} + j / k_{y2}) \quad (13)$$

$$G_{zz} = (\omega u_0 / \Delta) (-j(k_1^2 - k_{zn}^2) \text{ctg}(k_{y1} b) / k_{y1} - (k_2^2 - k_{zn}^2) / k_{y2}) \quad (14)$$

$$\Delta = (j(k_1^2 - k_x^2) \text{ctg}(k_{y1} b) / k_{y1} - (k_2^2 - k_x^2) / k_{y2}) (-j(k_1^2 - k_{zn}^2) \text{ctg}(k_{y1} b) / k_{y1} - (k_2^2 - k_{zn}^2) / k_{y2}) - k_x^2 k_{zn}^2 (\text{ctg}(k_{y1} b) / k_{y1} + j / k_{y2})^2 \quad (15)$$

We now use the Galerkin's method to solve the equations (10) and (11). To this end, the unknowns components of the current,  $J_{xn}$  and  $J_{zn}$ , are first expressed in terms of known base functions  $\xi_{pn}$  and  $\eta_{qn}$

$$J_{xn} = \sum_{p=1}^M c_p \xi_{pn} \quad (16a)$$

$$J_{zn} = \sum_{q=1}^N d_q \eta_{qn} \quad (16b)$$

where  $c_p$  ( $p=1,2,\dots,M$ ) and  $d_q$  ( $q=1,2,\dots,N$ ) are unknown coefficients. The base functions  $\xi_{pn}$  and  $\eta_{qn}$  are chosen to be the Fourier transforms of space-domain functions  $\xi_p(z)$  and  $\eta_q(z)$  which are nonzero only on the strip. Now substituting (16) into (11) and (12) and taking inner products of the resulting equations with base functions  $\xi_{pn}$  and  $\eta_{qn}$ , we obtain

$$\sum_p [c_p k_{ip}^{xx} + d_q k_{iq}^{xz}] = 0, \text{ for } i=1,2,\dots,M \quad (17)$$

$$\sum_p [c_p k_{jp}^{zx} + d_q k_{jq}^{zz}] = 0, \text{ for } j=1,2,\dots,N \quad (18)$$

where the coefficients are defined in terms of the scalar product of the base functions.

For the systems of linear homogeneous equations, (17) and (18), the condition for the existence of a nontrivial solution is that the determinant of the coefficient matrix vanishes, namely:

$$\det(K) = 0 \quad (19)$$

where  $K$  is the coefficient matrix. Elements of the matrix  $K$  are all functions of longitudinal propagating constant  $k_z$ ; therefore (19) is the equation to determine the allowed values of  $k_z$  and it will be simply referred to as the dispersion relation for the periodic structure.

#### NUMERICAL RESULTS

Due to the finite width of the antenna, the modes supported by the present antenna are hybrid in nature. The modal fields generally possess all six components which has been properly taken into account in our analysis. As the wave propagates progressively along the antenna it is scattered by the periodically loaded strips, resulting in the leakage or radiation of energy into the upper space. In the presence of radiation, the eigenvalue of the structure becomes complex. The real part determines the radiation angle. The imaginary part represents the rate of energy leakage and is thus referred to as the leakage constant. For the  $n$ -th spatial harmonic, the radiation angle of the beam<sup>(4)</sup> measured from broadside is given by the formula

$$\sin \theta_n = k_{zn} / k_0 \quad (20)$$

Fig. 2 shows the curve of leakage constants as a function of frequency for the strip width  $w = 0.20d$  ( $d$  is the period). It is found that the leakage constant of the  $TE_1$  mode is much greater than that of the  $TM_1$  mode which results from the fact that more energy concentrates near the metal strip region for the  $TE_1$  mode. It is noted that the leakage constants of the  $TE_1$  mode does not vary with frequency as rapidly as that of the  $TM_1$  mode. At the frequency around 41.2 GHz the phase velocities of the fundamental and 2nd space harmonics  $TE_1$  mode are nearly equal, and the mode-coupling<sup>1</sup> occurs which makes the leakage negligible. The same effect occurs at frequencies 37.1 GHz for  $TM_1$  mode. In addition of these well known effects, we obtain the new physical effect

which arising from the TM-TE polarization coupling at around 39.3 GHz, as shown in Fig. 2.

The effects of metal strip on the radiation characteristics of the antenna is illustrated in Fig. 3. As the strip width increases, the leakage constant reaches a maximum value and then decreases in certain frequency region. For the present structure the optimum ratio of  $w/d$  for maximum leakage lies between 0.35 and 0.40 for  $TE_1$ -mode.

#### EXPERIMENT

To verify the theory given in this paper an experiment is designed. The experimental model is scaled up to X-band for convenience. The model antenna, using the organic glass as its dielectric material, is such designed as to have the same cross-section with the waveguide BJ-100 (22.86mm x 10.16mm). The period is chosen to be 19.00mm and the strip width  $0.70d$ . A trough waveguide with metal flares attached to each side is used, in order to reduce the scattering at the metal edges. The experimental and the calculated radiation angles are both shown in Fig. 4, where the solid line is for the calculated values while the cross points for the experimental ones. Evidently, the experimental results agree well with the theoretical values.

#### ACKNOWLEDGMENT

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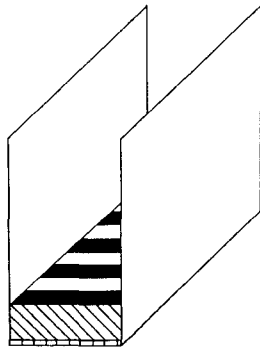


Fig.1 The strip-loaded millimeter wave integrated antenna

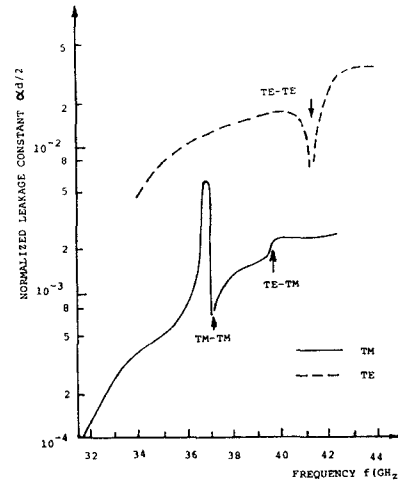


Fig.2 Normalized leakage constant versus frequency

(  $a=7.112\text{mm}$ ,  $b=3.556\text{mm}$ ,  $d=5.80\text{mm}$ ,  $w/d=0.20$ ,  
 $\epsilon_r=2.55$ ,  $k_x=\pi/a$  )

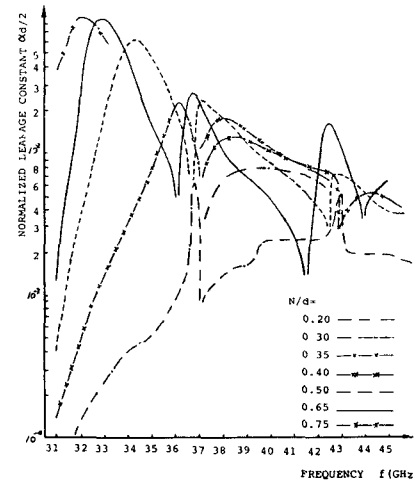


Fig.3 Normalized leakage constant versus frequency for

different strip widths for  $TH_0$ -mode (  $a=7.112\text{mm}$ ,  
 $b=3.556\text{mm}$ ,  $d=5.80\text{mm}$ ,  $\epsilon_r=2.55$ ,  $k_x=\pi/a$  )

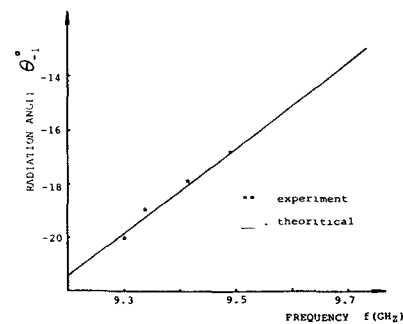


Fig.4 Comparison of the experimental and the theoretical results of the radiation angle (  $a=22.86\text{mm}$ ,  
 $b=10.16\text{mm}$ ,  $d=19.0\text{mm}$ ,  $w/d=0.70$ ,  $\epsilon_r=2.55$ ,  $k_x=\pi/a$  )